Lab 10: Standing Waves (M10)

Objectives

- Study the standing sound waves in adjustable air column.
- Observe and measure how the frequency of the string vibrations changes as the function of the length and the tension of the string.
- Study the fundamental frequency of a transverse standing wave.

Descriptions of waves

Sound - longitudinal waves usually produced by the vibrations of material objects. Sound waves need a medium (gas, liquid, or solid) to propagate.

Period - the time interval required for a wave to travel the distance of one wavelength.

Frequency - the number of wave crests arriving at a specific point per second. The equation for frequency is $f = 1/T$, where T is the period of the wave. The chromatic musical scale is determined by the frequency of the waves produced. The audible sound range for humans is from 20 Hz to 20 000 Hz (= 20 kHz). 1 Hz = 1 s^{-1} = 1/s.

Wave Crest - the highest point on the wave pattern.

Wave Trough - the lowest point on the wave pattern.

Amplitude - the maximum distance from the undisturbed position, measured either to the wave crest or from the undisturbed position to the wave trough.

Wavelength - the length of one cycle of the wave. The wavelength can be measured by finding the distance between two successive crests, two successive troughs, or any two successive equivalent points on the wave. The relation between sound wavelength (λ), velocity (v), and frequency (f) is given by: $\lambda = vT = v/f$.

Sound Velocity - the velocity of a traveling disturbance that represents sound. In air v_{air} = 343 m/s at 20 ˚C (room temperature) and depends on the air temperature (e.g., v_{air} = 331 m/s at 0˚C). Sound also propagates in liquids and solids. For example: v_{water} = 1500 m/s, v_{steel} = 5850 m/s, $v_{acrylic}$ = 2730.

Noise – a complex sound having numerous frequencies or tones, all of which have similar intensity. Its wave pattern looks like randomly generated jumps up and down.

Beats - the periodic variations in intensity at a given point due to the superposition of two waves having slightly different frequencies. When the two waves are sound waves, the variations in amplitude cause the loudness to vary at the beat frequency, which is equal to the absolute value of the difference between the frequencies of the waves:

$$f_{beats} = |f_2 - f_1| \tag{1}$$

Transverse Standing Wave – a pattern of vibrations that results when oppositely traveling waves of the same frequency and amplitude pass through each other. The direction of the vibrations is perpendicular (transverse) to the direction of motion of the original traveling waves.

Nodes – places on a transverse standing wave that do not vibrate.

Antinodes – places on a transverse standing wave where the maximum vibration occurs. The number of antinodes on a given wire is given by $n = \frac{2L}{\lambda}$, where L is the length of wire and λ is the wavelength.

Natural frequency – The frequency at which a system would oscillate on its own (or put more simply, the frequency or frequencies which the system finds most convenient for oscillation). The natural frequencies are determined by properties of the system, e.g., size, shape, tension.

Resonant frequencies – under resonant conditions, standing waves can be established only at certain frequencies f_n, known as the resonant frequencies. For a string that is fixed at both ends and has a length L, the resonant frequencies are:

$$f_n = n\frac{v}{2L} = nf_1 \quad \text{where} \quad f_1 = \frac{v}{2L} = \text{ fundamental frequency} \tag{2}$$

where v is the speed of the wave on the string and $n = 1, 2, 3,\ldots$ Please note that the velocity of the wave on the string is different from (usually much smaller) the sound velocity in the material from which the string was made! The vibration frequency corresponding to $n = 1$ (i.e., f_1) is called the **fundamental frequency** or the **first harmonic**. The vibration modes corresponding to $n = 2, 3, \ldots$ are called **second harmonic**, **third harmonic**, and so on. Frequencies above the fundamental are sometimes referred to as **overtones**.

Standing waves in air column – when we have air in a narrow tube, the sound waves propagate along the tube forming an air column going back and forth. Standing waves in the tube originate from interference (combination) of two sound waves travelling in opposite directions. One wave comes from a small speaker and the opposite wave is the reflected wave from the end of the tube.

A standing wave has **nodes** – points where the air inside the tube does not vibrate and **antinodes** – points where the amplitude of the air vibration is at maximum. Reflection of sound waves in the tube occur at both closed and open ends. When the end of the tube is closed, the air cannot freely vibrate, so a node must exist at the closed end. If the end of the tube is open, there is (almost) no resistance to air vibration and we have antinode there.

Resonance – A standing wave occurs when a wave is reflected from the end of the tube and the return wave interferes with the original wave. These sound waves will be reflected multiple times and travel between the end of the tube back and forth. In general, theses multiple reflected waves will not be in phase and the amplitude of the standing wave will be small. However, at certain frequencies, all reflected waves are in phase, creating a very high amplitude of standing wave vibrations. For these frequencies, the sound is the loudest and we call them resonant frequencies. The frequencies at which the resonance occurs depend on whether the ends of the tube are open or close.

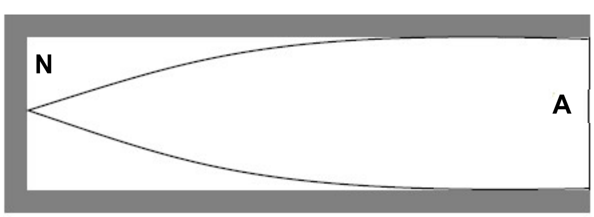
A. For a tube with length L, which is **open on both ends**, resonance occurs when the wavelength of the wave λ satisfies the following condition:

$$L = n * \lambda/2 \;, n = 1, 2, 3, \dots \; or \; L = \lambda/2 \;, \; \lambda, 3\lambda/2, \; 2\lambda, \dots$$

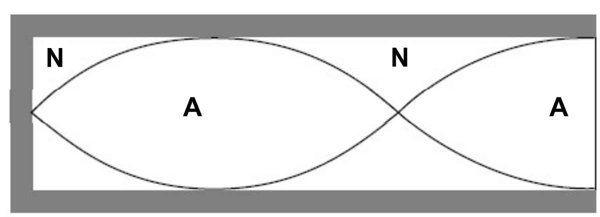
B. For a tube with length L, which is **closed on one end**, resonance occurs when the wavelength of the wave λ satisfies the following condition:

$$L = (2n-1) * \lambda/4 \;, n = 1, 2, 3, \dots \; or \; L = \lambda/4 \;, \; 3\lambda/4, \; 5\lambda/4, \dots$$

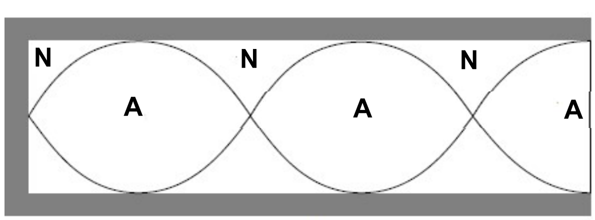
Resonance modes – The first four resonance modes for a tube **closed on one** end are shown below. In our experiment, the tube will be closed by a movable piston. The first resonance mode for n = 1 is called the fundamental. Successive resonance modes (n = 2, 3, 4, …) are called overtones (second, third, fourth, etc.). Nodes are marked with **N** and antinodes are marked with **A** *(PASCO Resonance Air Column – Instruction Manual)*.



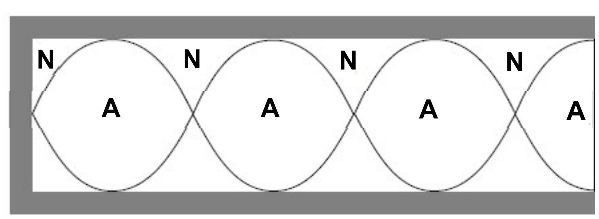
Closed Tube Fundamental



First Overtone



Second Overtone



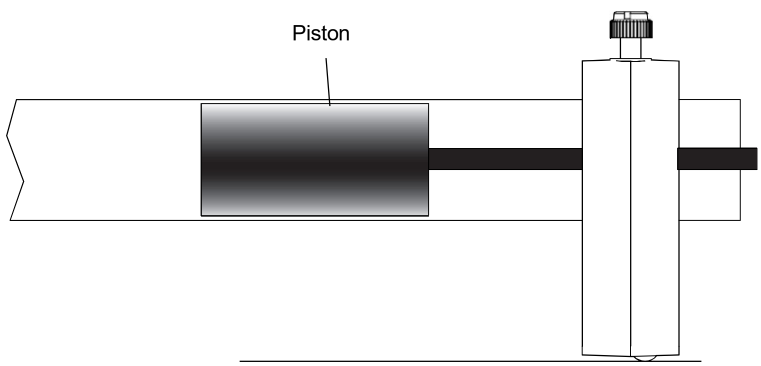
Third Overtone

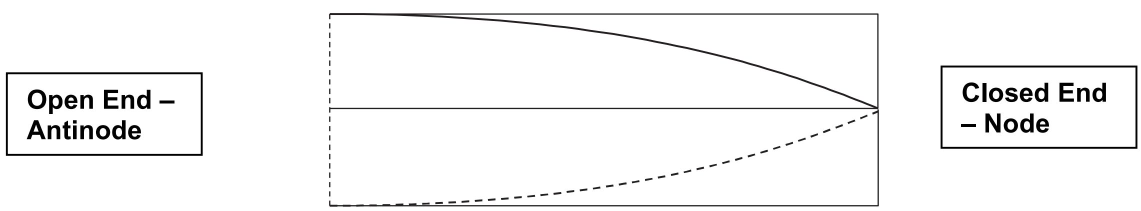
Procedure:

In Activity 1 we search for acoustic resonance condition inside an adjustable tube that is closed on one end. Activities 2 and 3 are about vibrations of a string with both ends at a fixed position.

Activity 1: Standing Sound Waves In a Tube (Closed On One End)

Download files required for experiment M10 from Brightspace for Physics 220. Double-click on the "*M10 Activity 1*". In Activity 1 we will use the Resonance Air Column with mini speaker located at the open end of the acrylic tube. The other end of the tube will be closed with the provided piston.





Place the Mini Speaker at an open end of the Resonance Air Column, with the front of the Mini Speaker about 1 cm from the open end of the tube. Make sure that the <u>speaker does not touch the tube</u>! Slide the piston inside the acrylic tube of the Resonance Air Column. Make sure that the frequency of the sinusoidal signal driving the mini speaker is 600 Hz and the amplitude is equal to 0.8 V. Set the meter stick on the table , next to the acrylic tube. **Precisely align the end (0 m) of the meter stick with the open end of the tube**.

At the open end of the tube, the sound wave shape depends slightly on the diameter of the tube. As a result of that special behavior, the <u>effective length</u> of the tube L_{eff} is slightly longer than the measured length. The size of the correction is proportional to the diameter of the tube d. Since the inside diameter of the tube is equal to d = 0.035 m, then for a tube closed on one end we have the following formula for effective length of the tube L_{eff}.

$L_{eff} = L + 0.3 * d = (2n - 1) * \lambda/4 \ , n = 1, 2, 3, \ldots \ or \ L = \lambda/4 \ , \ 3\lambda/4, \ 5\lambda/4, \ldots$ or

$L_{eff} = L + 0.01 \ m = (2n - 1) * \lambda/4 \ , n = 1, 2, 3, \ldots \ or \ L = \lambda/4 \ , \ 3\lambda/4, \ 5\lambda/4, \ldots$

Start with the piston inside the tube and very close to the speaker. Start recording. In this activity, we will use PASCO interface box to drive the mini speaker without any measurements. At this moment, you should hear a relatively quiet 600 Hz sound coming from the mini speaker. Next, slowly start moving the piston away from the mini speaker. It means that the length of the air column increases. You should hear the sound getting significantly louder. Find the position of the piston (i.e., the length of the air column) that corresponds to the maximum loudness (= resonance). At this position of the piston, the standing wave inside the air column will have an antinode at the open end of the tube. Record the length of the air column and continue slowly to the next resonance position.

Record the length of the air column the first three resonance positions on the data sheets.

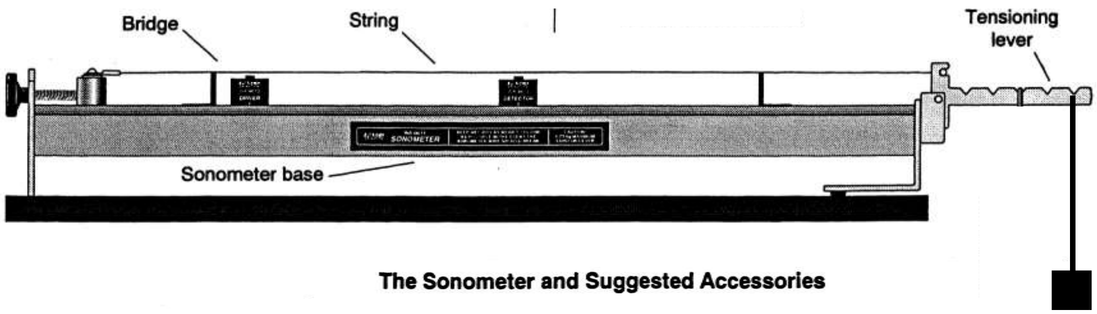
Using f = 600 Hz and the speed of sound in air at room temperature v_{air} = 343 m/s, calculate the theoretical length L_{theory} of the air column that corresponds to the first three resonances (antinodes at the tube open end. Calculate the difference between the effective length of the tube L_{eff} and the theoretical value of the length L_{theory}.

Next, change the frequency of the acoustic signal to 900 Hz. Repeat the measurements the same way as you did for 600 Hz sound. If the sound seems too loud, you may reduce the signal amplitude to 0.5 V.

Click on "**Stop**" to turn off the mini speaker. Exit Capstone and do not save any changes.

Activity 2: The Fundamental Frequency vs. the Tension of the String

For Activities 2 and 3 we will use the device called **sonometer** and we observe the frequency of mechanical waves on a single string. The sonometer looks like an electric guitar, but with only one string. Open "*M10 Activities 2-3*". On the sonometer, make sure that the two bridges, the black metal brackets supporting the string, are located at the 10- and 70-centimeter marks. The detector coil should be positioned in the middle of the sonometer (at the 40-cm mark). Carefully hang a 1.00 kg mass on the notch of the lever arm the **farthest away** from the sonometer. The wire may break if too much tension is put on the lever arm at once, so do not apply more weight than that from the 1.00 kg mass. Make sure that the tension lever arm is horizontal! See Fig. 2 for more explanation. If it is not, turn the screw on the left side of the sonometer to level it. Also, make sure that the mass is not swaying back and forth.



The Sonometer and Suggested Accessories

Fig. 1.

Check if the length of the vibrating part of the string *L* is set to 60 cm, i.e., that the supporting black metal brackets are at positions "10 cm" and "70 cm".

The tension force *F* of the wire should be at the last position on the lever arm is equal to $F = 5$ Mg, where M is the mass used and g = 9.80 m/s^2. Gently pluck the wire with the guitar pick at the center, close to the detector, as seen in Figure 1a. Clicking on the "**Start**" icon, collect the sound wave data for that frequency. When the sine waves have died down and become smooth, click the "**Stop**" button to "freeze" the oscilloscope display. Use the "Smart Tool"  icon to read the positions of two points separated by one period of oscillations (e.g., two adjacent maximums or minimums). Calculate the period and the fundamental frequency f_1 of oscillations.

<u>Move the weight one notch in (see Fig. 2 for more explanation) and find the new fundamental frequency</u>. Record the tension and frequency in the table provided in the data sheets. Remember that the **tension is a force,** not mass. Also, calculate the square root of the tension $\sqrt{F}$ and write it in the table.

Each time you move the weights <u>make sure that the lever arm is horizontal</u>. If it is not, turn the screw on the left side of the sonometer to level it.

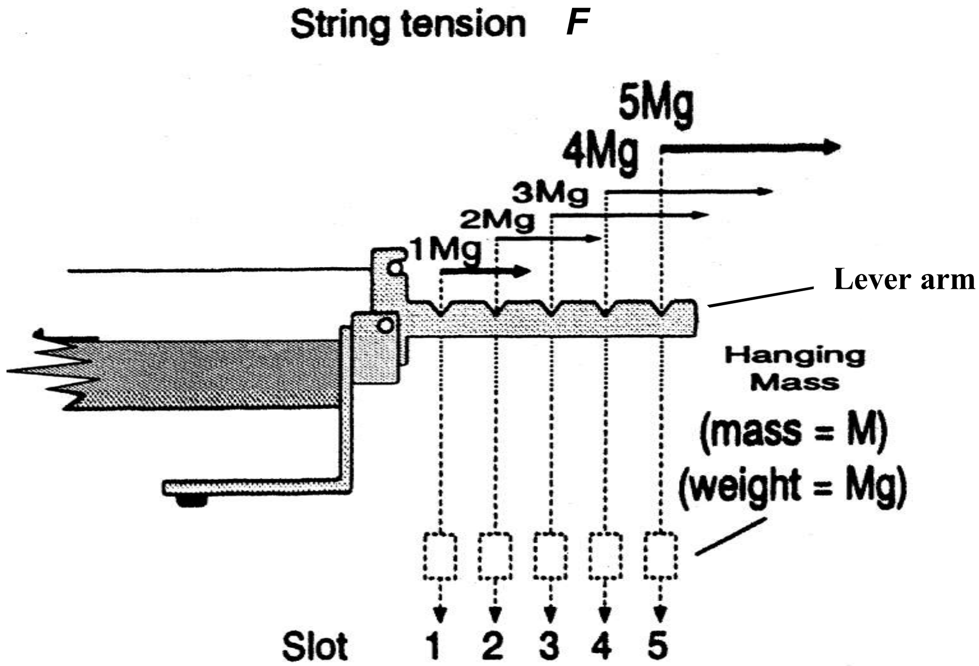


Fig. 2. Setting the tension[1]

The first frequency you find will be the highest fundamental frequency out of the five. As you move the weight in (closer to the suspension point), the tension force will decrease as the picture above suggests. As the tension decreases, the wire's amplitude may increase past the amplitude that should be associated with a certain fundamental frequency. At this point, the wire may strike the detector and then return to equilibrium amplitude.

Record the fundamental frequencies for each given tension. The theoretical formula for the fundamental frequency f_1 of a string is given by:

$$f_1 = \frac{v}{2L} = \left(\frac{1}{2L}\right)\sqrt{\frac{F}{\mu}} \quad , \quad v = \sqrt{\frac{F}{\mu}} \quad , \tag{5}$$

where F – tension force recorded, v is the velocity of the wave on the string and μ is the linear density of the string (μ = mass of the string m divided by its length L). The linear density of the string used in this experiment is equal to: $\mu = 1.84*10^{-3}$ kg/m. The length L is the length of the vibrating part of the string. For this Activity: $L = 0.70 - 0.10$ m $= 0.60$ m.

Using the theoretical equation for f_1, calculate the expected values for the fundamental frequency for given tensions. Are these values similar to the measured f_1? If not, then re-evaluate

[1] Sonometer – Instruction Manual by PASCO Scientific, 1988, p. 11.

your data. Find what the fundamental frequency would be if there were no tension. This will be only for graphing purposes.

Plot the measured fundamental frequency f_1 vs. $\sqrt{F}$. Draw the best-fit line (do **not** just connect the points!). Be sure to include the units. It is recommended that you use a computer-graphing program (e.g., MS Excel that is available in all ITaP labs). Use the 'linear fit' or "trendline" option to obtain the value of the slope of the best-fit line. **Print** this graph and attach it to this report. Write your name and those of your partners on the graph.

Activity 3: Frequency of a String as a Function of Its Length

In this Activity, we will measure the changes of the fundamental frequency with length of the string. We will use a fixed value of the tension: F = mg = 3.0 kg*9.8 m/s^2 = 3.0*9.8 N = 29.4 N. Make sure that the lever arm is horizontal. If it is not, turn the screw on the left side of the sonometer to level it. You can change the length of the vibrating part of the string L by moving the two black metal brackets supporting the string.

For a string, theory predicts that the frequency of vibrations is inversely proportional to the length of the vibrating part of the string.

$$f_x = \frac{v}{2L_x} = \frac{L_y}{L_x} f_y \quad \text{or} \tag{6}$$

$$\frac{f_x}{f_y} = \frac{L_y}{L_x} \tag{7}$$

where f_x is the frequency, the speed of the wave on the string is v, and L_x is the active length of the vibrating part of the string. Note that the speed of the wave on a string v is different from the speed of sound in the air. The speed of the wave on a string v depends on the thickness of the string and the applied tension force.

Check if the length of the vibrating part of the string L is set to 60 cm, i.e., that the supporting black metal brackets are at positions "10 cm" and "70 cm". Gently pluck the sonometer string and measure the frequency of vibrations using the same method as in *Activity* 2.

Change the length of the vibrating part of the string by moving the two black metal brackets supporting the string. Measure the frequency for the new length.

Calculate the f_x/f_{60} using:

(a) measured values of the wave frequency,

(b) the string frequency f_{60}, the ratio L_{60}/L_x and Equation 7.

Find the percentage error between the measured (a) ratio of f_x/f_{60} and the ratio calculated using Equation 7.

When finished, set the length of the vibrating part of the string L to 60 cm, i.e., that the supporting black metal brackets are at positions "10 cm" and "70 cm". Quit *Capstone*. Do not save any changes.

Complete the lab report and return it to the lab TA.

Make sure to complete the following tasks:

You must submit the answers to the prelaboratory questions online. (3.5 points)

1. Your completed Data Sheets. (6 points)
2. One graph from *Activity 2*. (0.5 points)
   (Title and write your name and those of your partners on each graph.)
3. Return the completed lab report to your lab TA.